Generalized Confluent Hypergeometric Systems included in Matrix Painlevé System

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Abstract

In this article, we investigate Generalized Confluent Hypergeometric Systems included in Matrix Painlevé Systems and corresponding gauge potentials of the Anti-Self-Dual Yang-Mills equations. Every nondegenerate Matrix Painlevé System $M_{\lambda}(k, l, m, n)$ includes a linear 2-system $LS_{\lambda}$ with respect to the variables $\mu$ and $\sigma$ and is equivalent to the Riccati equation $R_{J}$ included in Painlevé System $S_{J}$. Let $\tilde{M}_{\lambda}(k, l, m, n)$ be the symmetric ASDYM equation under the action of $PH_{\lambda}$ which is equivalent to the nondegenerate Matrix Painlevé System $M_{\lambda}(k, l, m, n)$. $M_{\lambda}(k, l, m, n)$ consists of the following equations:

\[
\begin{align*}
(1) & \quad \partial_{z} \Phi_{w} - \partial_{w} \Phi_{z} + [\Phi_{w} \Phi_{z}] = 0 \\
(2) & \quad \partial_{\tilde{z}} \Phi_{w} - \partial_{w} \Phi_{\tilde{z}} + [\Phi_{w} \Phi_{\tilde{z}}] = 0 \\
(3) & \quad \partial_{\tilde{z}} \Phi_{\tilde{z}} - \partial_{\tilde{w}} \Phi_{z} - \partial_{w} \Phi_{\tilde{w}} + \partial_{w} \Phi_{w} + [\Phi_{\tilde{w}} \Phi_{\tilde{w}}] - [\Phi_{w} \Phi_{\tilde{w}}] = 0 \\
(4) & \quad \partial_{\kappa} \Phi_{\kappa} = 0, \partial_{\tilde{\kappa}} \Phi_{\kappa} = 0, \partial_{\tilde{\kappa}} \Phi_{\kappa} = 0 \quad (\kappa = \tilde{z}, w, \tilde{w}, z)
\end{align*}
\]

$LS_{\lambda}$ is equivalent to a linear subsystem $\overline{LS_{\lambda}}$ in $\tilde{M}_{\lambda}(k, l, m, n)$. $\overline{LS_{\lambda}}$ consists of 8 equations which are classified into 3 groups (A) (B) (C). (A)(B) are included in (4) of $\tilde{M}_{\lambda}(k, l, m, n)$, while (C) is included in (1)(2)(3) in $M_{\lambda}(k, l, m, n)$.

Keywords

Generalized Confluent Hypergeometric System, Matrix Painlevé System, Anti-Self-Dual Yang-Mills equation