Uncertainty of the Keynesian Multiplier in Chile:

Empirical Evidence from Stochastic Simulation

December 2009

Shinji Yoshioka*

Abstract

In the last quarter of 2008, the world economy fell into a deep recession and many governments, of both developed and developing countries, took fiscal measures to stimulate the economy. The Government of Chile employed a sizable fiscal expansionary policy ahead of other developing countries. It is definitely essential to understand the size of the Keynesian multiplier and the GDP gap when adopting fiscal measures to stabilize or boost the economy. Therefore, this study focuses on Chilean expansionary fiscal policy implemented in 2009 by adopting concepts of the Keynesian multiplier and its uncertainty. Both deterministic and stochastic simulations, based on a small-sized IS-LM model, are performed. The results of the former reveal the essence of the textbook-style multiplier, and those of the latter, based on Monte Carlo simulations, indicate the uncertainty about the size of multipliers. This finding supports the Lucas critique, and requires a more precisely designed fiscal policy.

Key Words: Econometric Model, IS-LM Analysis, Fiscal Policy, Keynesian Multiplier, Monte Carlo Simulation

JEL Classification: C32, C51, E17, E62, H31

* Shinji Yoshioka, Faculty of Economics, Nagasaki University, s-ysk@nagasaki-u.ac.jp
The author is deeply grateful to Ms. Fujima, Secretary of Japanese Embassy in Santiago, Chile, for providing useful information on Chile's fiscal policy and economic situation.
1. Introduction

Since late 2008, the world economy has fallen into a deep recession caused by the US subprime bubble collapse and its aftermath, a financial crisis. In both developed and developing countries, strong fiscal measures have been taken to boost the economy and to cover the large negative GDP gap to a sizable extent. For example, figure 1.9 on p.16 of IMF (2009a) estimates that in 2010 the GDP gap in Latin America will increase to around 4 percent. Recently, in many countries, fiscal measures, rather than monetary policies, have therefore been employed to attain economic stabilization and stimulation. According to Hacienda (2009a) and Hacienda (2009b), the Government of Chile takes the following package, which amounts to US$ 4 billion to boost the country's economy.

1) Stimulation of investment and demand (US$ 750 million increase in public investment, and temporary elimination of the stamp tax, US$ 628 million in 2009, etc.)

2) Improved access to financing for business (temporary reduction of 15 percent in withholding tax for small and medium businesses, and of 7 percent for larger businesses, up to US$ 460 million, and increase of US$ 50 million in the CORFO\(^1\) financing facility for bank and non-bank factoring firms, etc.)

3) Economic support for individuals (special one-time payment of Ch$ 40 thousand (US$ 64) per dependent for beneficiaries of the Chile Solidario system and recipients of the Subsidio Único Familiar (SUF), etc., and early reimbursement of the 2010 income tax returns for individuals, etc.)

4) Promoting employment and training (creation of a subsidy to encourage formal employment for low-income young workers between 18 and 24, etc.)

In the face of copper price shocks originating in volatile foreign demand, Chile adopted a structural balance fiscal rule. Marcel C. et al. (2001) develop the structural budget balance methodology that has been adopted for use in the Chilean public sector since 2000. Kumhofl and Laxton (2009) also analyze this rule, using IMF's Global Integrated Monetary and Fiscal Model (GIMF), and conclude that a 0.5% surplus target, introduced in 2008, is desirable from a business cycle perspective. Fiess (2004) also stresses the countercyclical role of Chilean fiscal rule in the context of Ehrlich-Becker insurance framework.\(^2\) OECD (2009) concludes that Chilean fiscal resilience in 2007 is 2.42 and the highest in Latin American countries as Figure 1 reports.
After breaking the fiscal rule and adopting stimulus, however, it is essential to measure the impact of the policy, in particular, the size of the Keynesian multiplier and of the GDP gap. For example, Restrepo and Rincón (2006) estimate structural VAR and structural VEC models to evaluate the fiscal policy impact on Chile and Colombia and note that Chilean policy is more effective than Colombian, since public finances are well under control. Cerda et al. (2006) also find empirical evidence of the effectiveness of Chilean fiscal policy, using structural VAR.

This study also considers Chilean fiscal policy, estimating the size of the Keynesian multiplier, by employing small-sized IS-LM model simulations. Among existing studies, Serven and Solimano (1991) construct, estimate, and simulate a macroeconomic model for Chile, to explore the effects of domestic policies and external shocks, including a balanced-budget fiscal expansion. Corbo and Tessada (2003) use both macroeconometric and nonstructural VAR models to disclose the Chilean growth accounting and dynamic forecast, etc. On the other hand, this study constructs a small IS-LM model to evaluate the uncertainty of fiscal multipliers by focusing on the consumption function, which is a key factor for the multiplier. Both deterministic and stochastic simulations are conducted to reveal the uncertainty about the size of Chilean fiscal multipliers.

First, it is helpful to confirm that simple and textbook-style Keynesian multipliers are calculated differently for the three types, regardless of whether there is a closed or open economy, or whether the government sector, which does or does not collect taxes, is include. The model is described as follows:

\[ MULT = \frac{1}{1-c} \]
(EQ-2) Closed economy including the government sector (tax revenue is endogenous)

\[
MULT = \frac{1}{1 - (1-t)c}
\]

(EQ-3) Open economy including the government sector (both tax revenue and imports are endogenous)

\[
MULT = \frac{1}{1 - (1-t)c + m}
\]

where
- \(MULT\) multiplier
- \(c\) marginal propensity to consume
- \(t\) marginal tax rate
- \(m\) marginal propensity to import

According to the textbook-style explanation, the multiplier in a closed and government-excluded economy seems the largest of the three, and its size declines as the model assumption is expanded to include the government sector collecting taxes, and to open the economy that introduces the foreign sector, particularly imports. This study then estimates and compares the size of the Chilean multiplier by simulating three types of models based on (EQ-1), (EQ-2), and (EQ-3). Finally, to reveal the uncertainty of the size of the Keynesian multiplier, three types of stochastic simulations are performed on the basis of a Monte Carlo methodology.

The contents of the paper are as follows. The introduction overviews the main fiscal policy adopted by the Government of Chile in 2009 and some existing literature on Chilean fiscal policy, and presents basic or textbook-style concepts of the Keynesian multiplier. The next section reveals the structure of a small-sized IS-LM model of Chile, which includes four structural equations, estimated using the two-stage least squares method, and two definitions. The third section reports the deterministic, i.e., non-stochastic, simulation results of the model, which indicate a tendency to decline the size of multipliers, according to the expanding model sectors such as the government and the foreign sectors. The fourth section shows the stochastic, i.e., Monte Carlo, simulation results, including some uncertainty in the consumption function in both the disturbance term and the estimated parameter for disposable income. The final section briefly concludes the paper and presents some remaining issues. EViews V6 is employed for both estimations and simulations.

2. Structure of the Small-Sized IS-LM Model

This study employs a small-sized IS-LM Keynesian model in which demand factors determine the output level, and the production sector is not included. The following equations in (EQ-4) depict the structural functions, while (EQ-5) reports the definitions. All structural functions are estimated for the period 1986-2008, and the two-stage least squares method is employed to avoid estimation bias, where constant term and one-year lagged explanatory variables, etc., are included as instrument variables. The data adopted in the model are at current prices on an annual basis, and taken from IMF (2009b) except for
tax rate, which is calibrated. $R^2$ and SE denote the adjusted R square and the standard error of the equation, respectively. The numbers within parentheses under the equations indicate the standard errors of each parameter. Table1 reports the list of model variables.

(EQ-4) Structural equations

(1) Consumption function

$$C = 1.209 + 0.264 \ (GDP - TAX) - 0.0108 \ R + 0.673 \ C_{-1} + ERROR$$

(1.219) (0.0668) (0.0378) (0.116)

$R^2 = 0.998$, SE = 0.0621

(2) Investment function

$$I = 0.623 + 0.280 \ (GDP + M) - 0.203 \ (GDP_{-1} + M_{-1}) - 0.0077 \ R + 0.459 \ I_{-1}$$

(1.240) (0.0908) (0.104) (0.0380) (0.229)

$R^2 = 0.977$, SE = 0.800

(3) Import function

$$M = -1.461 + 0.358 \ GDP$$

(0.609) (0.0138)

$R^2 = 0.968$, SE = 1.677

(4) Money demand function

$$R = 25.123 + 6.868 \ log(GDP) - 10.826 \ log(M3)$$

(14.180) (9.366) (6.270)

$R^2 = 0.641$, SE = 6.501

(EQ-5) Definitions

(5) Gross domestic product

$$GDP = C + I + II + G + X - M$$

(6) Tax revenue

$$TAX = GDP * RTAX$$

Table1: List of Model Variables in Alphabetical Order

<table>
<thead>
<tr>
<th>Notation</th>
<th>Type</th>
<th>Definition</th>
<th>Unit</th>
<th>IFS code</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>endogenous</td>
<td>household consumption</td>
<td>billion pesos</td>
<td>22896F..ZF...</td>
</tr>
<tr>
<td>$G$</td>
<td>exogenous</td>
<td>government consumption</td>
<td>billion pesos</td>
<td>22891F..ZF...</td>
</tr>
<tr>
<td>GDP</td>
<td>endogenous</td>
<td>gross domestic product</td>
<td>billion pesos</td>
<td>22899B..ZF...</td>
</tr>
<tr>
<td>$I$</td>
<td>endogenous</td>
<td>gross fixed capital formation</td>
<td>billion pesos</td>
<td>22893E..ZF...</td>
</tr>
<tr>
<td>$II$</td>
<td>exogenous</td>
<td>changes in inventories</td>
<td>billion pesos</td>
<td>22893I..ZF...</td>
</tr>
<tr>
<td>$M$</td>
<td>endogenous</td>
<td>imports of goods and services</td>
<td>billion pesos</td>
<td>22898C..ZF...</td>
</tr>
<tr>
<td>M3</td>
<td>exogenous</td>
<td>money supply, M3</td>
<td>billion pesos</td>
<td>22859MC.ZF...</td>
</tr>
<tr>
<td>$R$</td>
<td>endogenous</td>
<td>lending rate</td>
<td>percent per annum</td>
<td>22860P..ZF...</td>
</tr>
<tr>
<td>TAX</td>
<td>endogenous</td>
<td>central government taxes revenue</td>
<td>billion pesos</td>
<td>228a11..CG...</td>
</tr>
<tr>
<td>RTAX</td>
<td>exogenous</td>
<td>rate of taxes on GDP</td>
<td>decimal</td>
<td>n.a.</td>
</tr>
<tr>
<td>$X$</td>
<td>exogenous</td>
<td>exports of goods and services</td>
<td>billion pesos</td>
<td>22890C..ZF...</td>
</tr>
</tbody>
</table>

Sources: Author and IMF (2009b)
Before proceeding to the model’s final test results, we must focus on the following four points:

1) The money demand function of equation (4) in (EQ-4) is set to be solved for the interest rate as an inverse function.
2) Since data for tax revenue at IMF (2009b) are available only for years 2005-2008 and their average proportion to GDP is 20.1 percent for four years, tax data are calibrated as 20 percent of GDP value and the tax definition equation (6) in (EQ-5) is set as proportional to GDP, which means both marginal and average tax rates are exogenously set at 20 percent before 2005, and \((GDP - TAX)\) is taken as a proxy for disposable income.
3) Imports and money demand functions are estimated on the basis of a normal and well-known assumption, and Yoshioka (2001) provides some background information for estimating the consumption and investment functions.
4) Since the Keynesian multiplier is defined in a short run, which means that prices are constant, all variables in the model are denoted as nominal.

A final test of the model is completed, and reports that the Root Mean Square Percentage Error (RMSPE) of GDP is around 0.40 percent. Figure 2 depicts both the actual GDP, and GDP as solved dynamically in the model. The model simulation results seem to have some tendency to overestimation in recent two years.

**Figure 2: Actual and simulated GDP in the final test of the model**

Note: The unit of the vertical axis is trillion Chilean pesos.
Source: Author’s estimation
3. Deterministic Simulation Results

Before proceeding to stochastic simulations, some deterministic simulations are performed. Here, "deterministic" means that neither the elements of uncertainty, nor those of stochasticity, are included in equations or parameters. According to (EQ-1), (EQ-2), and (EQ-3), three types of simulation are performed to confirm the decline in the multipliers when the sectors are included in a model. The model is simulated through its estimation period 1986-2008 with constant terms adjusted to the actual values of each variable for baseline simulations. A sustainable shock of 1 percent of GDP is added to government consumption for the final three years, from 2006 to 2008, to provide an impact simulation. Evidently, the multipliers are derived with the deviation between baseline and impact simulations. Table 2 reports the details of the model assumptions for estimated functions. Therefore, the smaller the number of cases, the larger the size of the multipliers is in a macroeconomics textbook.

Table 2: Model Assumptions

<table>
<thead>
<tr>
<th>Functions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption function</td>
<td>endogenous</td>
<td>endogenous</td>
<td>endogenous</td>
</tr>
<tr>
<td>Investment function</td>
<td>endogenous</td>
<td>endogenous</td>
<td>endogenous</td>
</tr>
<tr>
<td>Tax revenue definition</td>
<td>exogenous</td>
<td>endogenous</td>
<td>endogenous</td>
</tr>
<tr>
<td>Import function</td>
<td>exogenous</td>
<td>exogenous</td>
<td>endogenous</td>
</tr>
</tbody>
</table>

Note: Tax revenue is treated as exogenous at its value for Case 1 and endogenous for Cases 2 and 3 while tax rate is exogenous in both cases.

Source: Author

While the imports function is not included in the model in Cases 1 and 2, it is included in Case 3. On the other hand, tax revenue is treated as exogenous at its value in Case 1 and endogenous in Cases 2 and 3, where the tax rate is exogenous. For all simulation cases, the money demand function\textsuperscript{5} is always treated as endogenous, which means that the LM curve is not flat. According to Friedman (1973), the slope of the LM curve determines the size of the multiplier, and Romer (2000) discusses the requirement of the LM curve in a Keynesian framework. Apparently, Case 1 corresponds to (EQ-1), which excludes the government tax revenue definition and assumes a closed economy, and Case 2 is equivalent to (EQ-2), where the closed economy leaks some expanding fiscal policy effect to taxes. Case 3, the open economy, spills the stimulus to both taxes and the foreign sector through imports. Most economists seem to regard that the multiplier of Case 1 is the largest of the three cases and that of Case 3 is the smallest, whereas Case 2 lies midway between Cases 1 and 3. This argument is empirically proven in this paper. Figure 3 depicts the Chilean fiscal multipliers of three cases in Table 2.

- 6 -
### Figure 3: Sizes of Multipliers

<table>
<thead>
<tr>
<th>Year</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>1.36</td>
<td>1.26</td>
<td>0.87</td>
</tr>
<tr>
<td>2007</td>
<td>2.12</td>
<td>1.87</td>
<td>1.22</td>
</tr>
<tr>
<td>2008</td>
<td>2.69</td>
<td>2.25</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Note: The unit of the vertical axis is an absolute number, since the values shown are of multipliers.

Source: Author’s estimation

According to tax definition and imports function reported in (EQ-4), the marginal propensity to import is around 0.36, while the calibrated marginal tax rate is 0.20. Hence, the leakage of fiscal stimulus to taxes and the foreign sector is large in size. This is also confirmed by actual data of GDP components shown in Figure 4, which depicts imports' and taxes' shares to GDP.

### Figure 4: Imports' and Taxes' Shares to GDP.

Note: The unit of vertical axis is percent to GDP.
4. Stochastic Simulation Results

In addition to the previous deterministic, i.e., non-stochastic, simulations of the model, a few Monte Carlo simulations are completed for Case 3 in Table 2 where both imports and taxes are treated as endogenous. As mentioned above, this study focuses on the uncertainty of the consumption function. Thus two types of stochastic elements are employed and three types of Monte Carlo simulations are executed as follows:

1) Case A includes the uncertainty in the entire consumption function, i.e., a disturbance term that is the product of the equation standard error given as $SE_1$ in ensuing equation in (EQ-6) and a normal random number is added at the equation's error term:

2) Case B includes the uncertainty in the parameter for the disposable income at the consumption function, i.e., a disturbance term that is the product of the parameter's standard error given as $SE_2$ in ensuing equation in (EQ-6) and a normal random number is added in the parameter for disposable income; and,

3) Case C includes both uncertainties in 1) and 2) above.

(EQ-6) Consumption Function Including Uncertainty

$$C = 1.209 + 0.264 \times (1 + SE_2 \times RAND_2) \times (GDP - TAX) - 0.0108 \times R + 0.673 \times C_{-1} + SE_1 \times RAND_1$$

where $SE_1$ Standard error for entire consumption function

$SE_2$ Standard error for parameter of disposable income

Here, normal random numbers are calculated from uniform random numbers according to Box-Müller transformation in Box and Müller (1958), which generates two sets containing 1000 random numbers that follow $N.I.D. (0,1)$. These two sets of normal random numbers are stored and identical random numbers are employed for all three cases, which is one of the most remarkable features of the Monte Carlo simulations in this paper. Figure 5 depicts the histograms of these two sets of normal random numbers. The sample means are almost equal to zero and the sample standard deviations are almost identical to one.
Figure 5: Histograms of Normal Random Numbers

(1) **RAND1**: Random Numbers for whole Consumption Function

Sample mean: -0.037227
Sample std. dev.: 1.02769

Sample mean: 0.0029948
Sample std. dev.: 0.98241

Source: Author's transformation

After completing the Monte Carlo simulations for each case 1000 times, the values of standard deviations of impact simulations in last three years for each case are derived from the results, and compared with deterministic simulation results of Case 3 in Table 2 and Figure 3. Figure 6 reports multipliers in each case. The center bars of each year depict the multiplier derived from deterministic simulation results of Case 3 in Table 2 and Figure 3, while the left-side and right-side bars indicate those to which is added or deducted, respectively, a value of one unit of the standard deviation derived from each Monte Carlo simulation case.
Figure 6: Multipliers Derived from Monte Carlo Simulations

(1) Case A: Including uncertainty in whole consumption function

(2) Case B: Including uncertainty in coefficient of disposable income at consumption function
(3) Case C: Including uncertainty both in whole consumption function and in coefficient of disposable income at consumption function

It is not difficult to see at a glance that the multiplier deviations of the stochastic simulation from each deterministic case grow larger as time passes, i.e., from 2006 to 2008. And deviations in case A, B, and C grow larger in this order, according to the amount of uncertainty included in the consumption function. One of the most impressive results derived from these simulation results should be stressed: The deviation of the parameter uncertainty case is far larger than that of the equation uncertainty case. This is partly due to small significance or large standard error of the parameter, but it also reminds the Lucas critique presented in Lucas (1976), which insists that the estimated parameters of models are not necessarily structural, i.e., not policy-invariant or policy-variant so that they may change whenever policy is changed. This also suggests that policy conclusions based on those models would therefore potentially be misleading. Since the model system of this study reported in (EQ-4) does not adapt to Lucas critique, the Monte Carlo simulation is employed for approximation. The estimation results reported Figure 6 reveal that the Keynesian fiscal multiplier may possibly show a large negative decline when the model includes some uncertainty in the consumption function and, in particular, in its parameter for disposable income, which is usually called the marginal propensity to consume.

5. Conclusion and Remaining Issues

Many economists consider that Romer and Bernstein (2009) were unveiled to provide an empirical basis on which to promote fiscal stimulus not only in the United States, but also all over the world, including Chile. On the other hand, some critical literature, including Cogan et al. (2009) and Davig and Leeper (2009), has also been presented, while Smets and Wouters (2007), adopting Bayesian DSGE approach, insist that the multiplier is around 1.0-0.4 and far smaller than that of Romer and Bernstein.
This study too is inspired by Romer and Bernstein (2009); however, it employs a different approach from other literature that adopts stochastic Monte Carlo simulations, which reveal that the Keynesian fiscal multiplier may become negative if sizable uncertainty is included in consumers' behavior and, in particular, in marginal propensity to consume. The uncertainty of the Keynesian multiplier emerges in both deterministic and stochastic simulations. Even in deterministic simulations, the Keynesian multiplier varies, depending on whether the government and foreign sectors are included in the model or not. Of course, stochastic simulation results unveil large uncertainty about the size of multipliers. From another point of view, the results of this study support Lucas critique, and suggest that more precisely designed fiscal policy is required in both developed and developing countries, since developing countries also utilize fiscal policy as autonomous tool for stabilizing national economy after the recent recession.

A few remaining issues should also be noted. First, more accuracy in equation estimations is required. Partly due to limited data availability of the general government's tax revenue in developing countries, the parameter for disposable income in the consumption function, which defines the marginal propensity to consume, may contain some degree of bias. Second, more comprehensive model is required. The model constructed in this paper consists of only four structural functions and two definitions. On the other hand, a more comprehensive model does not necessarily bring more precise simulation results. Third, quarterly models, including the model built in Romer and Bernstein (2009), are employed in most developed countries. In developing economies, however, it is difficult to obtain seasonally adjusted quarterly data. It is not easy to resolve these remaining issues quickly, but further efforts to investigate developing areas will be made.

(Endnote)

1 CORFO is Production Development Corporation, a government entity for industrial promotion in Chile. For further information, see its web site (http://www.corfo.cl/).
2 See Ehrlich and Becker (1972).
3 Romer and Bernstein (2009), at p.12, suggest that the size of multiplier based on Government purchases for the US is around 1.5-1.6.
4 In the government-excluded economy here, both government expenditure and tax revenue are treated as exogenous, and the word "government-excluded" does not mean the absence of the government or its expenditure.
5 The money demand function in the model is solved for an interest rate function as a reverse function. This seems usual approach in econometric models.
6 For this calculation, EXCEL is employed.
7 Hence the Monte Carlo simulations completed in this paper are reproducible.
(References)


Chambers, John and Moritz Kraemer (2005), Sovereign Report Card: EMBIG, Commentary Report, Standard & Poor's

(Accessed on November 13, 2009)


Corbo, Vittorio and José A. Tessada (2003), Growth and Adjustment in Chile: a Look at the 1990s, Central Bank of Chile Working Papers No.204, Central Bank of Chile, March 2003

Davig, Troy and Eric M. Leeper (2009), Monetary-Fiscal Policy Interactions and Fiscal Stimulus, NBER Working Paper No.15133


Hacienda (2009a), Gobierno lanza plan de estímulo fiscal equivalente a 2,8 puntos del PIB para potenciar la economía y defender el empleo, Ministerio de Hacienda, Gobierno de Chile [in Spanish]


Hacienda (2009b), Plan de estímulo fiscal, Ministerio de Hacienda, Gobierno de Chile [in Spanish]

(Accessed on November 13, 2009)

IMF (2009a) World Economic Outlook – Crisis and Recovery, International Monetary Fund


OECD (2009) Latin American Economic Outlook 2010, Development Center, Organisation for Economic Co-operation and Development
Restrepo, Jorge E. and Hernán Rincón (2006), Identifying Fiscal Policy Shocks In Chile and Colombia, Central Bank of Chile Working Papers No.370, Central Bank of Chile

Romer, Christina and Jared Bernstein (2009), The Job Impact of the American Recovery and Reinvestment Plan
http://otrans.3cdn.net/ee40602f9a7d8172b8_ozm6bt5oi.pdf (Accessed on November 13, 2009)


http://tsq.link.net.id/paper.htm (Accessed on November 13, 2009)