【長崎大学経済学部ディスカッション・ペーパー・シリーズ】
DISCUSSION PAPER SERIES
Faculty of Economics, Nagasaki University
No.2014-05 (March 2014)

Generalized Confluent Hypergeometric Systems included in Matrix Painlevé System

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Abstract

In this article, we investigate Generalized Confluent Hypergeometric Systems included in Matrix Painlevé Systems and corresponding gauge potentials of the Anti-Self-Dual Yang-Mills equations. Every nondegenerate Matrix Painlevé System $M_\lambda(k,l,m,n)$ includes a linear 2-system LS_λ with respect to the variables μ and σ and is equivalent to the Riccati equation R_J included in Painlevé System S_J. Let $\tilde{M}_\lambda(k,l,m,n)$ be the symmetric ASDYM equation under the action of PH_λ which is equivalent to the nondegenerate Matrix Painlevé System $M_\lambda(k,l,m,n)$. $\tilde{M}_\lambda(k,l,m,n)$ consists of the following equations:

$$\begin{cases} (1) \quad \partial_z\Phi_w - \partial_w\Phi_z + [\Phi_z\ \Phi_w] = 0 \\ (2) \quad \partial_{\tilde{z}}\Phi_{\tilde{w}} - \partial_{\tilde{w}}\Phi_{\tilde{z}} + [\Phi_{\tilde{z}}\ \Phi_{\tilde{w}}] = 0 \\ (3) \quad \partial_z\Phi_{\tilde{z}} - \partial_{\tilde{z}}\Phi_z - \partial_w\Phi_{\tilde{w}} + \partial_{\tilde{w}}\Phi_w + [\Phi_z\ \Phi_{\tilde{z}}] - [\Phi_w\ \Phi_{\tilde{w}}] = 0 \\ (4) \quad \partial_p\Phi_\kappa = 0, \partial_q\Phi_\kappa = 0, \partial_r\Phi_\kappa = 0 \qquad (\kappa = \tilde{z}, w, \tilde{w}, z) \end{cases}$$

LS_λ is equivalent to a linear subsystem $\widetilde{LS_\lambda}$ in $\tilde{M}_\lambda(k,l,m,n)$. $\widetilde{LS_\lambda}$ consists of 8 equations which are classified into 3 groups (A) (B) (C). (A)(B) are included in (4) of $\tilde{M}_\lambda(k,l,m,n)$, while (C) is included in (1)(2)(3) in $\tilde{M}_\lambda(k,l,m,n)$.

Keywords
Generalized Confluent Hypergeometric System, Matrix Painlevé System, Anti-Self-Dual Yang-Mills equation