Money and Good, Liquidity and Acceptability

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ABSTRACT

This paper analyzed the differences between the single money economy and the multi-money economy where all goods in the economy perform the function of medium of exchange. We concluded that single money economy is the ultimate outcome of the process wherein the less liquid goods are excluded in turn. The most impressive finding is that moneyness, the total exchange feasibility of the economy, does not change during this exclusion process. However, the economy can attain a more efficient situation when one good supports the total moneyness and all other goods can be consumed physically or put into the production process.

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1. INTRODUCTION

This paper attempts to answer the question : What is the substantial difference between the single money economy and the money-goods economy, where money and near-money coexist ? Needless to say, the modern economy is the money-goods economy because we have a series of monetary devices,cash, deposit, IOU, credit and so. To consider this problem we hereby introduce three concepts deeply related with one another : liquidity, acceptability and moneyness.

2. MONETARY USE AND PHYSICAL USE

First of all, we consider the nature of a good in the exchange economy. All goods can be consumed physically as well as monetarily (economically) in the sense that we use them as the medium of exchange. Take gold for instance. We can make some ornaments by using gold as the raw material. On the other hand, it can be used as the medium of exchange . Needless to say, the extent to which a good is regarded as the exchange medium deeply depends on the social custom or history, however, theoretically we can say all goods has the potential of being used both for monetary use and for physical use.

Here we can write the potential use of good i as

$$(1,\ 1)_i$$

where the first (second) 1 shows that the good has the potential monetary(physical) use. For the good not used as the medium at all, j , we have

$$(0,\ 1)_j$$

and for the good not consumed physically(=used only as the exchange medium) , k, we have

$$(1,\ 0)_k$$

By using these expressions we get

$$(1,1)_1 + (1,1)_2 = (1,0)_1 + (0,1)_1 + (1,1)_2$$

The left-hand side shows the economy where all the goods are used for monetary and physical use .On the other hand, in the economy depicted by the right hand side, the 1st good is the only medium of exchange. Intuitively speaking, the left-hand side economy is the barter economy because all good can be traded with one another by receiving their worth in the form of good. In the exchange economy where a good can be acquired by paying the medium of exchange , the 1st good corresponds to the right hand side. Historically our society has evolved from the barter economy to the exchange economy, i.e.,from the left-hand side to right hand side.

If we assume the potential use of the goods is additive, the total benefit which can be derived from the goods is same (=2) in both economies,

$$(2\ ,\ 2)_{\ 1,\,2\,=\,\text{all goods}}\ .$$

What has happened in this transition process ? This is the key question we will try to answer in succeeding sections.

3. BASIC CONCEPTS

Our discussion focuses on the monetary use of goods. Here we assume that monetary use has two characteristics inherent in the good itself, liquidity and acceptability. The *liquidity* of *i*-th goods (denoted by l_i) is defined as the ability to describe the credit/debt relationship. In the other words, at the transaction, if one can use some good as the measuring device to determine how much one owes the other, that good has got *liquidity*. For example, if you can say that you owe your friend "three desks" when you buy something from him, these desks have some liquidity.

Suppose there are m goods in the economy and n goods of m have liquidity, we can line them up in decreasing order.

$$l_1 > l_2 > l_3 > \cdots l_n. \qquad l_i>0, \quad n<m \tag{1}$$

However, your friend may not cancel your debt by taking three desks. Hence we need to define the *acceptability* of *i*-th good as the medium of exchange (denoted by a_i). *Acceptability* is defined as the possibility that one accepts some good in compensation for the goods he has given the others. The liquidity and acceptability depend on not only the good's character but also on the social custom and tradition . Without loss of generality, we could assume

$$a_1 + a_2 + \ \dots + a_n = 1 \tag{2}$$

In the exchange economy where some good is used as a medium of exchange, that good is said to have the *moneyness* or the agents can enjoy *the benefit from the monetary use.* Here we define the moneyness of *i*-th good (m_i) as follows:

$$m_i = a_i\, l_i$$

This means the moneyness is an increasing function of liquidity and acceptability. People can use any good they prefer as the medium of exchange so the moneyness of the economy is

$$M = a_1 l_1 + a_2 l_2 + \ldots + a_n l_n \tag{3}$$

It should be noted that (2) does not mean that the public can use all good as the exchange medium in transaction. Instead, it only shows the maximum moneyness the economy can provide.

The visual expression of moneyness is shown in FIG-1 and FIG-2.

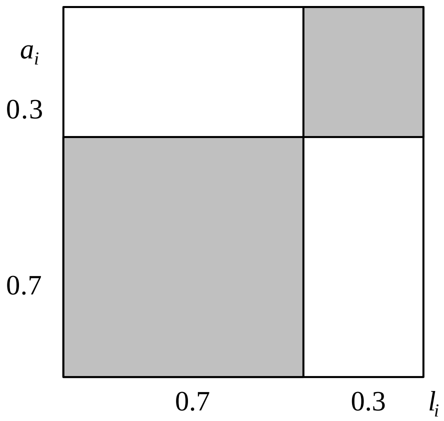

FIG-1

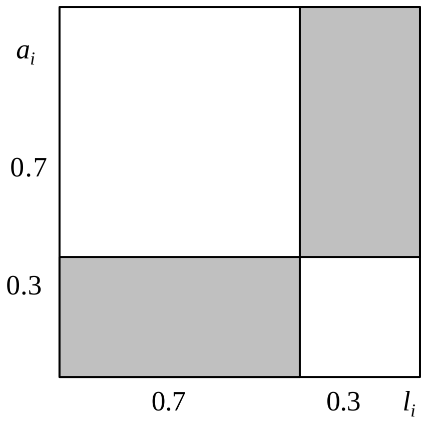

FIG-2

When $a_1=0.7$, $a_2=0.3$, $l_1=0.7.l_2=0.3$, the shadowed area in FIG-1 represents the moneyness. FIG-2 shows the case of $a_1=0.3$, $a_2=0.7$, $l_1=0.7.l_2=0.3$. The upper-left area in FIG-1, for example, shows the loss from the fact that 30% of all agents never use 1st good with l_i=0.7.

The 1st good with l_1=0.7 and a_1=0.7 dominates all other good in terms of liquidity and acceptability. However, what if the 1st good is characterized by l_i=0.7 and a_1=0.3 ? The 1st good cannot dominate the 2nd good because the two shadowed areas are equal. Which then is the medium of exchange ? Both ?

Now we are in the stage to define *money.* To do so, we need the next theorem on the moneyness and acceptability.

Theorem 1 : M has maximum value if and only if $a_1 > a_2 > a_3 \cdots > a_n$.

Proof **:**

(sufficiency: $a_1 > a_2 > a_3 \cdots > a_n \Rightarrow \max M$)

STEP 1. the case of n = 2..

M is

$$M_1 = a_1 l_1 + a_2 l_2. \qquad (4)$$

Here M', where the order of *a* does not correspond to *l*'s order, is

$$M_1' = a_2 l_1 + a_1 l_2. \qquad (5)$$

We have, by assumptions,

$$M_1 - M_1' = (a_1 l_1 + a_2 l_2) - (a_2 l_1 + a_1 l_2) = (l_1 - l_2)(a_1 - a_2) > 0 \qquad (6)$$

Accordingly M_1 is maximum in $n=2$ economy.

STEP 2. the case of n = 3.

The ordering of *li* is given. First, we consider the case where l_1 corresponds to a_1. Then (l_1 l_2 l_3) must correspond to (a_1 a_2 a_3) or (a_1 a_3 a_2). Then our problem is reduced to a comparison of

$$M_2 = a_2 l_2 + a_3 l_3 \text{ and } M_2' = a_3 l_2 + a_2 l_3 \quad . \qquad (7)$$

$M_2 > M_2'$ is directly concluded in the above fashion. Accordingly, $M_3 = a_1 l_1 + M_2 > a_1 l_1 + M_2'$. This implies M_3 is the maximum in n=3 economy. For the case where l_2, ,l_3 corresponds respectably to a_2 ,a_3, the same discussions can be applied.

STEP 3 : In the case of $n=4$, assuming the same analogy as the above, we can easily derive that if $l_1 > l_2 > l_3 > l_4$ and $a_1 > a_2 > a_3 > a_4$, M_4 is the maximum.

STEP 4 : In the same fashion, we find in the case of $n = k$. $l_1 > l_2 > l_3 \cdots > l_k$ and $a_1 > a_2 > a_3 \cdots > a_k$ and in the case of $n = k+1$. $l_1 > l_2 > l_3 \cdots > l_{k+1}$ and $a_1 > a_2 > a_3 > \cdots a_{k+1}$, M_k,M_{k+1} are the maximums. This completes the proof.

(necessity: $\max M \Rightarrow a_1 > a_2 > a_3 \cdots > a_n$)

In $n=2$ economy, because $M_1 = a_1 l_1 + a_2 l_2$ is maximum then

$$M_1 = a_1 l_1 + a_2 l_2 > M_1' = a_2 l_1 + a_1 l_2.$$

and

$$M_1 - M_1' = (a_1 l_1 + a_2 l_2) - (a_2 l_1 + a_1 l_2) = (l_1 - l_2)(a_1 - a_2) > 0 \qquad (8)$$

$l_1 > l_2$ leads immediately to $a_1 > a_2$. In $n>2$ economy the same discussion yields the same conclusion, thus yielding *Theorem* 1.

Because *Theorem* 1 guarantees that the situations like FIG-2 can not emerge, it is rational to define money as follows.

Definition: Money is the good with the highest liquidity and the highest acceptability.

4 LIQUIDITY CHANGE

We now turn to the case where the *i*-th good's liquidity changes. Starting from a simple case, we first study *Theorem* 2.

Theorem 2 : Even if the i-th good is monetised, the economy can not enjoy the increase in the total moneyness.

Proof: Suppose that the liquidity of the *i*-th good is increased in the fashion, for simplicity, by $l_1 = l_i + \alpha$. Suppose the *i*-th good is the desk then $l_1 = l_i + \alpha$ means the desk are sold for money(=*monetised*). The total moneyness could be written as

$$M_{after} = l_1 a_1 + l_2 a_2 + \cdots + (l_1 - \alpha) a_i + \cdots + l_n a_n$$

$$=(a_1+a_i)l_1 + l_2 a_2 + \cdots + l_{i-1} a_{i-1} + l_{i+1} a_{i+1} + \cdots + l_n a_n - \alpha a_i \tag{9}$$

It must be noted that the *i*-th good is not used as the medium of exchange. Instead, the acceptability of money is increased by (a_1+a_i). On the other hand, the economy incurs the cost $(-\alpha a_i)$, the loss of moneyness originating from the *i*-th good . Then we have

$$M_{after} - M = (l_1 - l_i - \alpha)a_i = 0. \tag{10}$$

Examining (10), we see that the total moneyness is not changed. Further, it is impossible to hold $l_i + \alpha > l_1$ because it violates the condition that 1st good is money. Therefore *Theorem* 2 is proved.

Theorem 2 states that the economy can enjoy the moneyness M without the i-th good if the 1st good's acceptability is increased. Of course the excluded i-th good is used as the physical good. Any other implications are given in Section 7 together with the implications of the next section.

5 SINGLE MONEY ECONOMY

The above procedure can be applied to derive the moneyness of the single money economy. We derive the single money economy by excluding the last good(n-th good) , n-1 th good , n-2 th good and so on, from the series of medium of exchange.

The liquidity change is assumed to be subject to

$$l_1 = l_2 + \alpha_2 ,\ l_2 = l_3 + \alpha_2 ,\ l_3 = l_4 + \alpha_3 \ \cdots \ l_{n-1} = l_n + \alpha_{n-1} \tag{11}$$

This means that the liquidity of i-th good is increased by α_{i-1} and equalized to the liquidity of the i-1 th good. We have

$$\begin{aligned} M_1 &= l_1 a_1 + (l_1 - \alpha_2)\ a_2 + (l_2 - \alpha_3)\ a_3 + \cdots + (l_{n-1} - \alpha_n)\ a_n \\ &= [\ l_1(a_1+a_2) + l_2 a_3 + l_3 a_4 + \cdots + l_{n-1} a_n] - [\alpha_2 a_2 - \alpha_3 a_3 - \cdots - \alpha_n a_n] \end{aligned} \tag{12}$$

(12) states the n-th good is not used as the medium of exchange. Instead the acceptability of the 1st good (=money) in increased with some costs(=2nd term in the right-hand side). Further *Theorem* 2 also holds because

$$\begin{aligned} M_1 - M &= [\ l_1(a_1+a_2) + l_2 a_3 + l_3 a_4 + \cdots + l_{n-1} a_n] \\ &\quad -[l_1 a_1 + l_2 a_2 + \cdots + l_n a_n] - [\alpha_2 a_2 - \alpha_3 a_3 - \cdots - \alpha_n a_n] \\ &= (l_1 a_2 - \alpha_2 a_2) + [l_1(a_3 - a_2) - \alpha_3 a_3] + \cdots + [l_{n-1}(a_n - a_{n-1}) -- \alpha_n a_n] - l_n a_n \\ &= a_2 (l_1 - \alpha_2) + [\ a_3(l_2 - \alpha_3) - a_2 a_3] + \cdots + [a_n (l_{n-1} - \alpha_n) - a_{n-1} l_{n-1}] - l_n a_n \\ &= l_2 a_2 + [\ l_3 a_3 - l_2 a_2] + [l_4 a_4 - l_3 a_3] + \cdots + [\ l_n a_n - l_{n-1} a_{n-1}] + l_n a_n \\ &= l_n a_n - l_n a_n = 0 \end{aligned} \tag{13}$$

Substituting (11) into (12) and arranging gives

$$\begin{aligned} M_2 &= [\ l_1(a_1+a_2+a_3) + l_2 a_3 + l_3 a_4 + \cdots + l_{n-2} a_n] \\ &\quad - [\alpha_2 (a_2+a_3) - \alpha_3 (a_3+a_4) - \cdots - \alpha_{n-1} (a_{n-1}+a_n) - \alpha_n a_n] \end{aligned} \tag{14}$$

and $M_2 = M_1$.

We found that the n-1 th good is excluded from the economy. Iterating the same calculation eventually results to

$$\begin{aligned} M_{n-1} &= l_1(a_1+a_2+a_3 \cdots a_n) \\ &\quad - [\alpha_2 (a_2+a_3 \cdots a_n) + \alpha_3 (a_3+a_4 \cdots a_n) + \cdots + \alpha_{n-1} (a_{n-1}+a_n) + \alpha_n a_n] \end{aligned} \tag{15}$$

(14) indicates the moneyness of the economy where only money is accepted as the medium of

exchange. Needless to say ,

$$M_{n-1} = M_{n-2} = \cdots = M \ , \tag{16}$$

the subscript represents the number of unused goods as the medium. Then we call the economy with M_{n-1} the *single money economy* and the economy with $M_k (k<n-1)$ the *money-goods economy* .

To make clear the difference between M_{n-1} and M_k, we assume $\alpha_i = \alpha_j \equiv \alpha$ for all i,j by changing the unit of l_i. Then we have

$$\begin{aligned} M_{n-1} &= l_1(a_1+a_2+a_3 \cdots a_n) - \alpha [(a_2+a_3 \cdots a_n)+(a_3+a_4 \cdots a_n) + \cdots + (a_{n-1}+a_n) + a_n] \\ &= l_1(a_1+a_2+a_3 \cdots a_n) - \alpha [a_2 + 2a_3 + 3a_4 + \cdots + (n-2) a_{n-1} + (n-1) a_n] \end{aligned} \tag{17}$$

Consider the case when all good except money is directly and simultaneously monetised, i.e.,

$$l_1 = l_2 + a = l_3 + 2a = l_4 + 3a = \cdots = l_n + (n-1) \alpha \tag{18}$$

It is easily confirmed that (17) can be derived by substituting (18) into *M.* This implies that the markets for all goods can be traded for money and contribute to improve the exchange feasibility.

Our method, where the last good is excluded one after another, describes the historical process that the physical mediums have been eliminated from the transaction. For instance take the shell that was used as the medium in ancient China. When more convenient good was recognized, the shell terminated its role as the medium.

It must be useful to provide a simple numerical example to illustrate the nature of (17).

EXAMPLE 1: $n=4, l_1=10, l_2=8, l_3=6, l_4=4, \alpha=2, a_1=0.4, a_2=0.3, a_3=0.2, a_4=0.1$

$M = (10\times0.4)+(8\times0.3)+(6\times0.2)+(2\times0.1) = 4 + 2.4 + 1.2 + 0.2 = 8.0$

$M_3 = 10 - 2 [(0.3+0.2+0.1)+(0.2+0.1)+(0.1)] = 10 - 2(1) = 8.0$

FIG-4 represents the moneyness of the money-goods economy. By sliding three upper rectangles in FIG-4 to the left we get FIG-3 for the single money economy. In FIG-3, the size of ① is 2×0.1 because $l_1-l_2=2$.Similarly, ②=2×(0.2+0.1),③=2×(0.3+0.2+0.1). Hence, $M_3=M$.

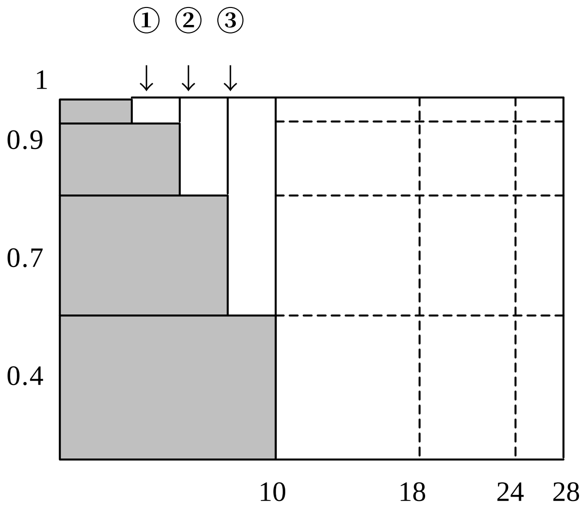

FIG-3(single money)

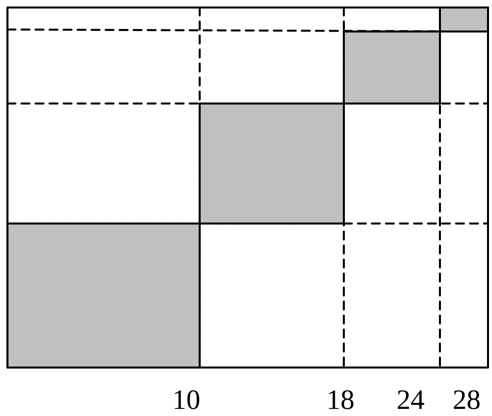

FIG-4(money-goods)

In Fig-4, all four goods are utilized for the monetary use as well as physical use. On the other hand, the economy depicted by FIG-3 assigns only the 1st good as the exchange medium and all the other goods are physically consumed.

What happens if n is increased, i.e., the n+1 good is added as the medium? If a_{n+1} is added,at least one a_k $(k< n)$ must be decreased so as to keep (2). It is natural to assume $k = n$ because $n+1$-*th* good is a close substitute for the n-th good as medium. Therefore,

$$a_{n+1}' + a_n' = a_n \tag{19}$$

where prime(') implies "after"the n+1-th good is added. Using (19), we obtain

$$M_{(n+1)-1} = l_1 a_1 + l_2 a_2 + \cdots + l_n a_n' + l_{n+1} a_{n+1}' , \quad l_n > l_{n+1} \tag{20}$$

and

$$M_{n-1} - M_{(n+1)-1} = l_n a_n - l_n a_n' - l_{n+1} a_{n+1}' = l_n(a_n - a_n') - l_{n+1} a_{n+1}' = a_{n+1}'(l_n - l_{n+1}) > 0 \tag{21}$$

(21) indicates that the moneyness is decreased as n is increased.

The added good takes the acceptability from the n-th good, however, its liquidity is smaller than the n-th good. So the moneyness lost is bigger than the gain.

On the other hand, if the acceptability of the new good is located between l_i and l_{i+1} $(i<n+1)$, the moneyness does not decrease. Suppose the new good has l_{new} $(l_i > l_{new} > l_{i+1})$. The moneyness after introducing the new good is

$$M'_{(n+1)-1} = l_1 a_1' + l_2 a_2' + \cdots + l_i a_i' + l_{new} a_{i+1}' + l_{i+1} a_{i+2}' + \cdots + l_n a_{n+1}'$$

where $a_i{}' > a_i' > a_{i+1}{}'$, $a_1{}' + a_2{}' + \ \ldots\ a_{n+1}{}' = 1$ (22)

If the acceptability is adjusted for all a_i like (21), we can not decide the sign of $M_{n-1} - M'_{(n+1)-1}$. For simplicity, we assume the newly introduced good takes the acceptability from l_{i+1}, i.e.,$a_i{}' = a_i$ *for all* $i<n$. In other words, condition (22) should be kept only by the last two terms is kept. As a result, we have

$$M_{n-1} - M'_{(n+1)-1} = -l_{new}\, a_{i+1} + (a_{i+1} - a_{i+2})\, l_{i+1} + (a_{i+2} - a_{i+3})\, l_{i+2} + \cdots + (a_{n-1} - a_n{}')\, l_{n-1} + (a_n - a_{n+1}')\, l_n$$

$$= -l_{new} a_{i+1} + (a_{i+1} - a_{i+2})\, l_{i+1} + (a_{i+2} - a_{i+3})\, l_{i+2} + \cdots + (a_{n-1} - a_n{}')\, l_{n-1} + (a_n')\, l_n$$

$$= a_{i+1}(l_{i+1} - l_{new}) + a_{i+2}(l_{i+2} - l_{i+1}) + \cdots + a_{n-1}(l_n - l_{n-1}) + a_n{}'(l_n - l_{n-1}) \qquad (23)$$

All those in parentheses are negative because of the nature of liquidity-order. It shows that moneyness is increased, although it depends on the acceptability profile.

Fig-5 and FIG-6 show the case whereby the newly good($l_2 > l_{new} > l_3$.) is added into the 3 money-goods economy. The moneyness is

$$M_2 = l_1 a_1 + l_2 a_2 + l_3 a_3 \quad \text{and} \quad M_{2+1} = l_1 a_1 + l_2 a_2 + l_{new} a_3{}' + l_3 a_4{}' \quad \text{where } a_3 = a_3{}' + a_4{}' \qquad (24)$$

The net gain, accordingly, is

$$M_{2+1} - M_2 = l_{new} a_3{}' + l_3 a_4{}' - l_3 a_3 = l_{new} a_3{}' + l_3 (a_4{}' - a_3) = a_3{}'(l_{new} - l_3), \qquad (25)$$

The reason behind this is simple. The vertical area in FIG-5 ($l_3 a_3$) represents the moneyness lost and the horizontal areas in FIG-6 ($l_{new} a_3{}'$) is the moneyness gained. The net gain is ① - ②, which equals to $a_3{}'(l_{new} - l_3)$, the last term of (23) or (25).

Next consider the case $m \rightarrow \infty$. FIG-7 and FIG-8 depict the case where $l_i - l_j = \alpha < \varepsilon$ (for all $i,j = i-1$, ε is a sufficiently small positive number). The moneyness of a barter economy is a concave-line because of *Theorem* 2 and that of a single money economy is the a-axis itself. This is the foundation of the widespreading knowledge that the evolution of the human economy is characterized by the process of improving the moneyness, i.e., the exchange feasibility, by conversing into a single medium of exchange and by introducing the new monetary device.

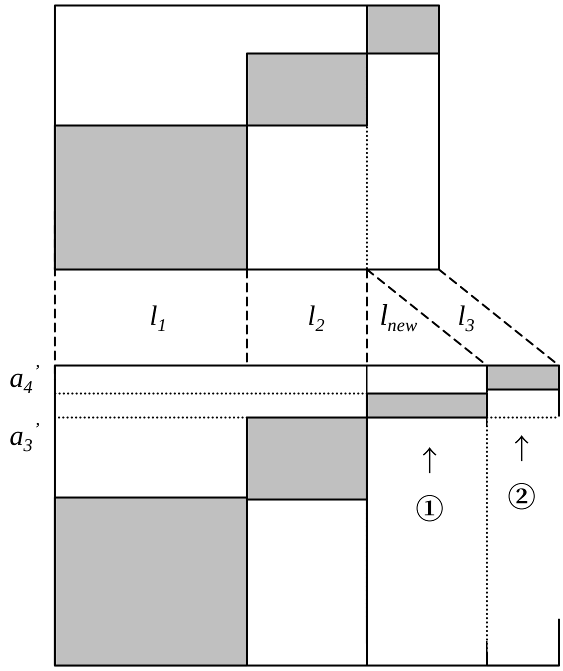

FIG-5(before)

FIG-6(after)

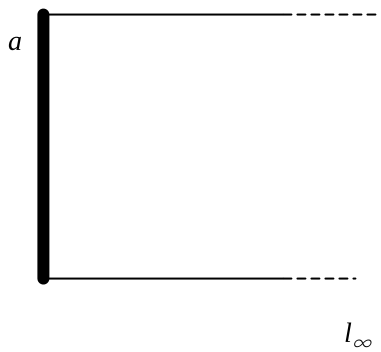

FIG-7(single money)

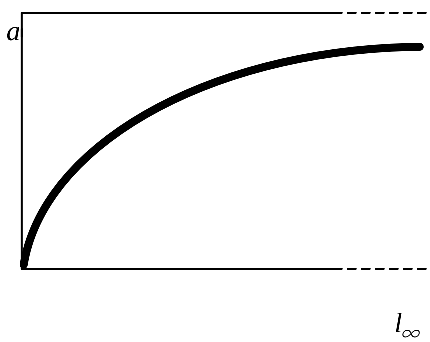

FIG-8(barter)

Now, suppose $n = m$. This leads us to the barter economy because all goods can be the exchange medium for all other goods. Therefore we know that *the monetary economy and barter economy are indifferent in the sense the moneyness of both economies is equal.*

6 DISCUSSIONS

In modern society, all goods are private property and can be freely traded with one other. This also means all goods can serve as the exchange medium if both trading parties agree. Simultaneously it should be noted that all goods have their own utilities if the owner consumes or utilizes them directly. Then we can refer to the former as the *monetary use* and the latter as the *physical use*.

The benefit of monetary use comes from our two concepts, liquidity and acceptability. In other words, moneyness, M, is the summation of the utility the economy can enjoy when all goods are in monetary use. The liquidity change discussed in section 4 is the process to distinguish the monetary use and the physical use then integrate the former into the money(1st good) and leave the latter to the good itself. As a result, the good used only in the physical sense is excluded from the list of monetary devices. However, it concentrates in the utilization of its physical use, for example, as raw materials.

Here is an implication for the asset-liquidation or securitisation. If the i-th good is the claim to the future income stream, the liquidity change in this case corresponds to the asset liquidation by securitisation. The above explanation shows that in asset liquidation, the monetary use of the i-th good is transferred into money and the economy can use the i-th good in physical use. This is the reason why the asset liquidation is expected to be the remedy for the non-performing loan. By securitising the non-performing loan(often backed by real estate), the economy can acquire the monetary use of the backed real estate.

7 CONCLUSION

The explicit purpose of this study was to examine several important concepts of monetary economics. If the moneyness could depend on the liquidity and acceptability of goods, the single money economy and the money-goods economy are indifferent in the sense their moneyness are equal.

It is interesting to examine the social effects of the asset-liquidation or securitisation. Given the number of goods, the asset-liquidation can not increase the moneyness.

The only way to improve the moneyness is the injection of a new device whose liquidity dominates one of the existing goods into the monetary system. In this sense, the liquidation of the existing goods works not as the creation mechanism of moneyness, but as the liquidity-allocation mechanism.

論文要旨：

物々交換経済に見られるように、すべての財は交換手段として用いられる可能性を持つ一方、財それ自体として消費されうる。それに対して、近代的な経済は特定の財のみが交換手段として機能する貨幣経済として特徴づけられる。物々交換経済はどのような過程を経て貨幣経済へて進化するのかを、すべての財に潜在的に含まれる交換手段としての２つの要素（流動性(liquidity)**、受容性**(acceptability)**）およびそれによって決定される経済全体の交換可能性（貨幣性**(moneyness)**）を中心に考察した。そして、ある財の流動性が何らかの理由によって高められ、それに伴い受容性が高まるという過程を想定することにより、単一の財が貨幣として機能する経済を論理的に再現した。この分析は、近年注目を集めている債権流動化の機能について興味深い含意をもたらす。**